1.1.2 Logic Puzzles

Tug of War!

Who will win the tug of war in round 3?

Round 1: On one side are four teachers, each of equal strength. On the other side are five students, each of equal strength. The result is dead even.

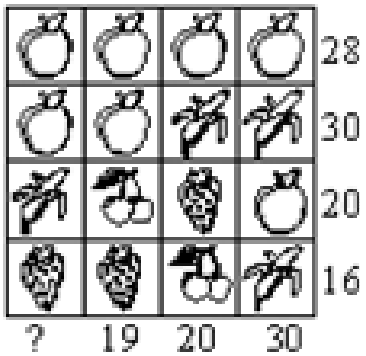
Round 2: On one side is Buddy, a dog. Buddy is put up against two of the students and one teacher. The result, once again is dead even.

Round 3: Buddy and three of the students are on one side and the four teachers are on the other side.

Who do you think will win the third round? Explain.

Puzzling Fruit

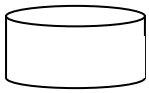
In the puzzle below, the numbers alongside each column and row are the total of the values of the symbols within each column and row. What should replace the question mark? Make sure you provide a full and detailed solution.



1.1.3 Buddy's Hungry

Buddy, one of the teacher's dogs, is very hungry. Ms. Jones stops at the pet store on her way home from school. She is always looking for the most economical buy. While at the pet store, she notices the following prices of pet food:

Five 150 mL cans of *Perfect Pet* dog food for $1.26
Twelve 400 mL cans of *Doggies Love It* for $7.38
Ten 150 mL cans of *Rover's Chow* for $2.60
Six 400 mL cans of *Man's Best Friend* for $3.94



Which pet food should Ms. Jones buy? Explain in as many different ways as possible.

1.2.2: Review of Metric Length Units

	milli-	centi-	deci-	base unit	deka-	hecto-	kilo-
distance	1000 mm (millimetre)	100 cm (centimetres)	10 dm (decimetres)	1 m (metre)	0.1 dam (dekametres)	0.01 hm (hectometres)	0.001 km (kilometres)
mass	1000 mg (milligrams)	100 cg (centigrams)	10 dg (decigrams)	1 g (gram)	0.1 dag (dekagrams)	0.01 hg (hectograms)	0.001 km (kilograms)
volume	1000 mL (millilitres)	100 cL (centilitres)	10 dL (decilitres)	1L (litre)	0.1 daL (dekalitres)	0.01 hL (hectolitres)	0.001 kL (kilolitres)

Complete the following:

1. Fill in the blanks below with the correct number.

   a) 1 m = _____ mm b) 1 m = _____ cm c) 1 cm = _____ mm

   d) 1 km = _____ m

2. Convert each given measurement to the unit specified.

   a) 4.5 m = _____ mm b) 5.3 m = _____ cm c) 25.8 cm = _____ mm

   d) 36.8 km = _____ m e) 5694 m = _____ km f) 2.5 mm = _____ cm

3. The diameter of a golf ball is about 4 cm. What is the radius of the ball in millimetres?

4. Fill in the blanks with the correct units

   a) 8 m = 8000_____

   b) 500 mm = 50_____

   c) 85____= 8500 cm

1.2.3 Metric Conversions

Complete the following conversions.

1. 1000 mL = _____ L
2. 120 mm = _____ cm
3. 1200 mL = _____ L
4. 2 cm = _____ mm
5. 11000 L = _____ kL
6. 10 cL = _____ mL
7. 12000 m = _____ km
8. 8 g = _____ cg
9. 80 ml = _____ cl
10. 3 L = _____ cL
11. 2000 L = _____ kL
12. 5 cm = _____ mm
13. 900 cm = _____ m
14. 11 cg = _____ mg
15. 9000 m = _____ km
16. 7000 mL = _____ L
17. 5 kg = _____ g
18. 60 mm = _____ cm
19. 1 kg = _____ g
20. 4000 mL = _____ L
21. 1 cL = _____ mL
22. 1100 cL = _____ L
23. 10000 g = _____ kg
24. 2000 mL = _____ L
25. 7000 L = _____ kL
26. 70 ml = _____ cL
27. 5 g = _____ cg
28. 9 cL = _____ mL
29. 1 g = _____ cg
30. 8 kg = _____ g
31. 6 g = _____ cg
32. 6 km = _____ m
33. 30 mg = _____ cg

1.2.3 Metric Conversions (Continued)

1. 3 metres = _____ centimetres
2. 40 litres = _____ dekalitres
3. 600 milligrams = _____ grams
4. 5 kilometres = _____ hectometres
5. 70 centimetres = _____ metres
6. 900 decilitres= _____ dekalitres
7. John's pet python measured 600 centimetres long. How many metres long was the snake?

8. Faith's mass was 3.8 kilograms at birth. What was her mass in grams?

9. Jessica drank 4 litres of tea today. How many decilitres did she drink?

10. Fill in the blanks with the correct units

    a) 10 km = 10000_____

    b) 50000 mm = 50_____

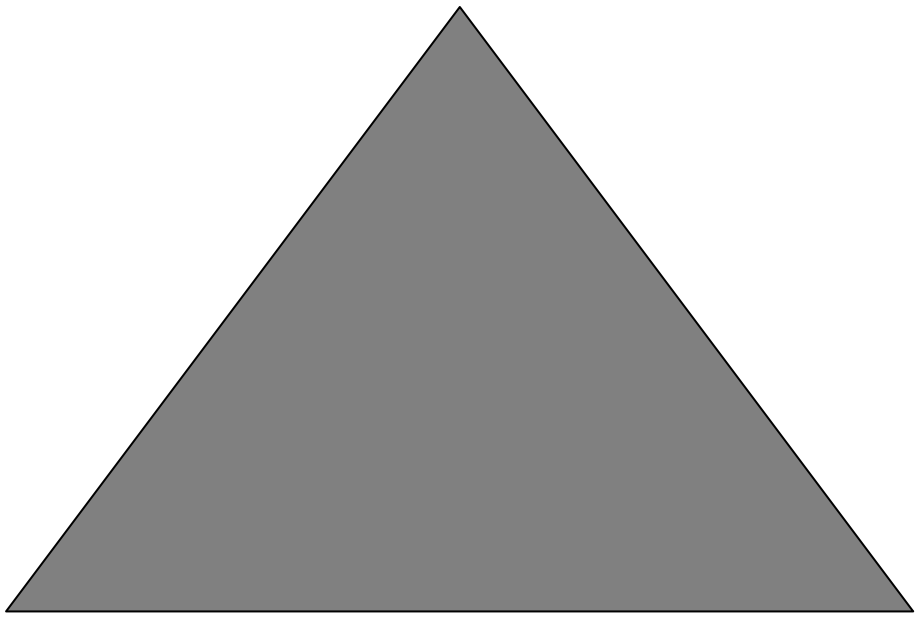
    c) 85____= 8500 cm

1.2.4 Growing Shapes

Problem: For the triangle drawn below:

a) Make another triangle that has exactly the same shape and whose perimeter is twice as long.
b) Make another triangle that has exactly the same shape and whose perimeter is half as long.
c) Determine the area of the three triangles (original, double, half)
d) Determine the relationship between the side length and the area of the triangle. For example, what happens to the area when side length is doubled?

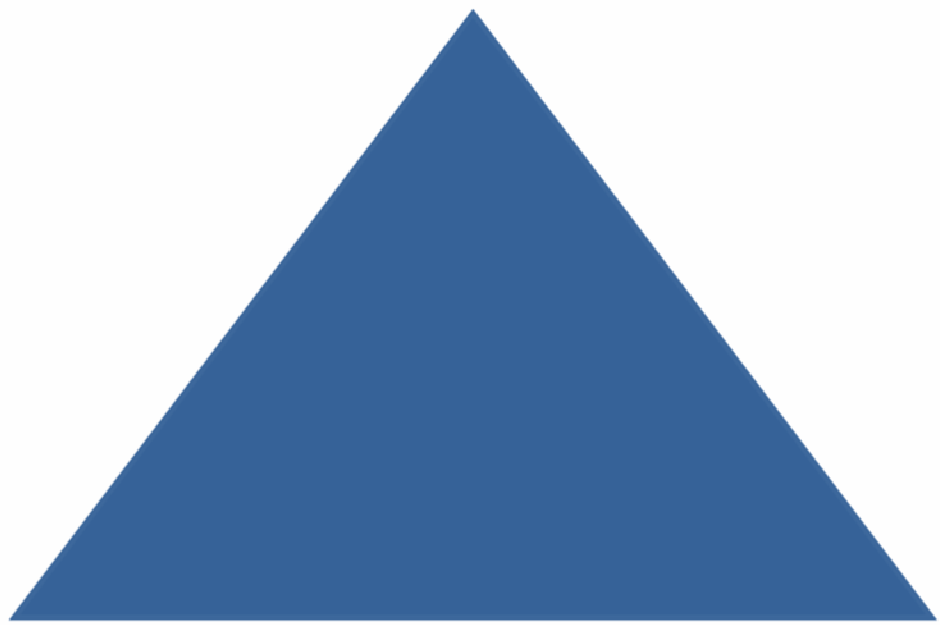
Show your work and reasoning in each case.



1.3.2: Growing and Shrinking Triangles

Investigation

Find the area and perimeter of the triangle.



If another triangle of the same shape has a perimeter that is double, what is the effect on the area? If another triangle of the same shape has a perimeter that is half, what is the effect on the area?

Hypothesis

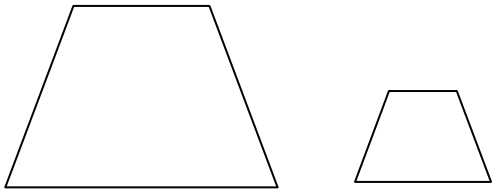
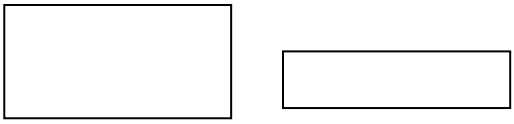
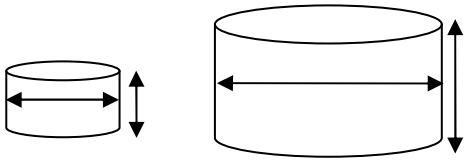
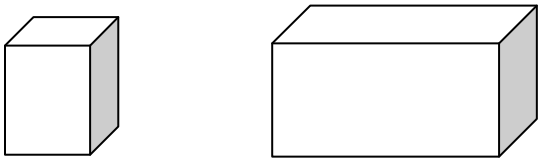
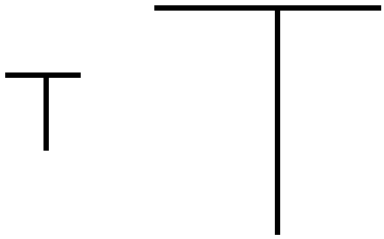
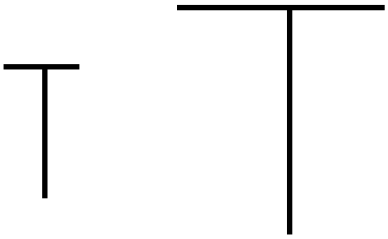
If one triangle of the same shape has double the perimeter of the original triangle, the resulting area of the triangle would be _____.

Complete the investigation.

Show your work and explain your reasoning. Generalize by stating the relationship between the perimeter and the area of similar triangles. State a conclusion based on your work. This conclusion may be based on your original hypothesis.

1.4.1 What is Similarity?

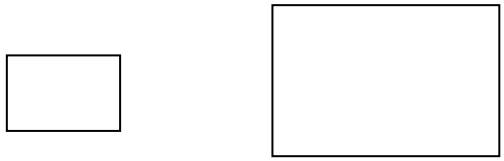
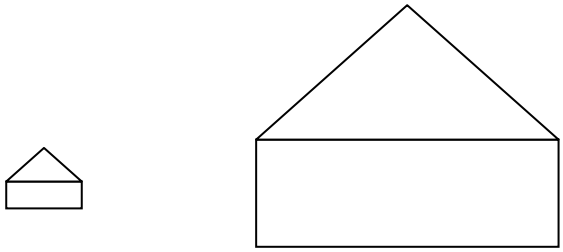
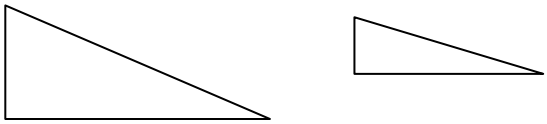
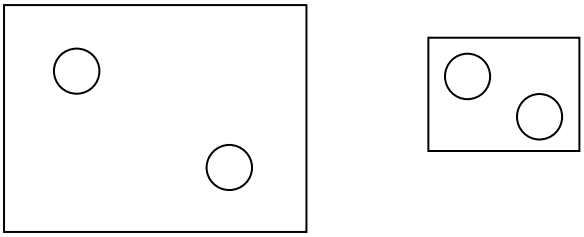
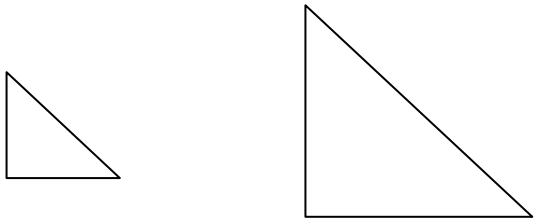
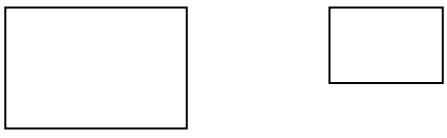
What does it mean if we say that 2 objects are similar? See if you can find out by using the clues below. *Hint: Use a ruler and a protractor to make measurements.*

Clue #1 These 2 objects are similar 	**Clue #2** These 2 objects are not similar 
Clue #3 These 2 objects are similar 	**Clue #4** These 2 objects are not similar 
Clue #5 These 2 objects are similar 	**Clue #6** These 2 objects are not similar 
Clue #7 These 2 objects are similar 	**Clue #8** These 2 objects are not similar 

What do you think similarity means?
Formal Definition of Similarity:

1.4.1 What is Similarity? (continued)

In each question, decide if the objects are similar (yes or no) and then explain:
Hint: Use a ruler and a protractor to make measurements.

	Similar? _____ Explain:
	Similar? _____ Explain:
	Similar? _____ Explain:
	Similar? _____ Explain:
	Similar? _____ Explain:
	Similar? _____ Explain:

1.4.2: What Is Similarity?

Anticipation Guide

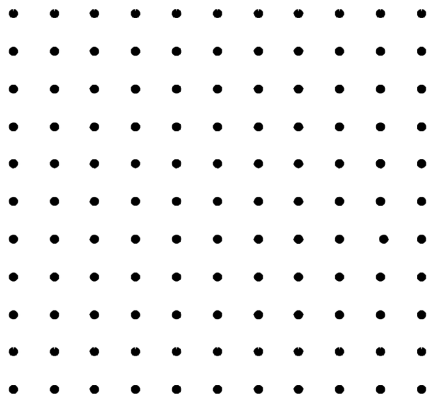
Before		Statement	After	
Agree	**Disagree**		**Agree**	**Disagree**
		In a triangle, I can calculate the length of the third side if I know the length of the other two sides.		
		All triangles are similar.		
		All squares are similar.		
		When I enlarge a geometric shape, the number of degrees in each angle will become larger.		

1.4.3: What Is Similarity?

1. a) On your geoboard create a right-angled triangle with the two perpendicular sides having lengths 1 and 2 units.

   b) Create two more triangles on your geoboard that are enlargements of the triangle created in a).

2. Draw the three triangles using different colours on the grid and label the vertices, as indicated:
   - triangle one (label vertices ABC)
   - triangle two (label vertices DEF)
   - triangle three (label vertices GHJ)



1.4.3: What Is Similarity? (continued)

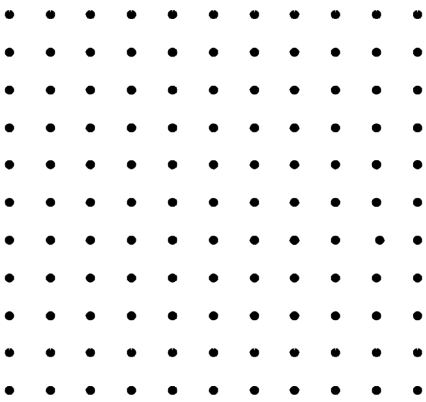
3. a) Determine the lengths of the hypotenuse of each of the :

(**Hint**: Pythagorean Theorem)

△ABC	△DEF	△GHJ

b) Indicate the length of each side of each triangle on the diagram.

4. a) Place △ABC, △DEF, and △GHJ on the geoboard so that one vertex of each triangle is on the same peg and two of the sides are overlapping.



b) Copy your model on the grid.

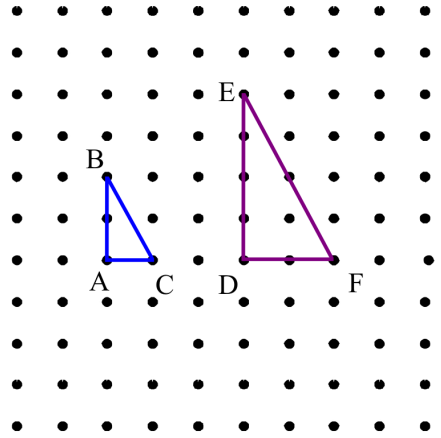
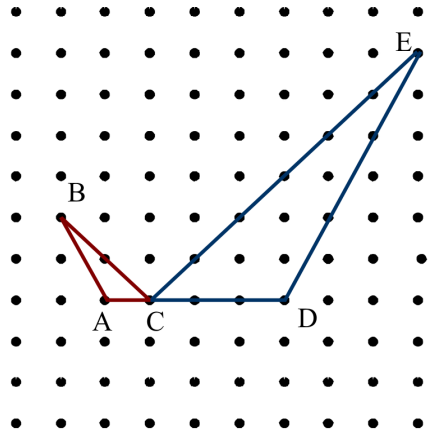
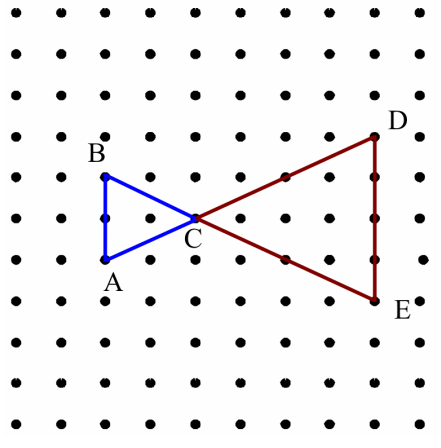
5. a) What do you notice about the corresponding angles of △ABC, △DEF, and △GHJ?

b) What do you notice about the corresponding sides of △ABC, △DEF, and △GHJ?

Summary: I know the following about similar triangles

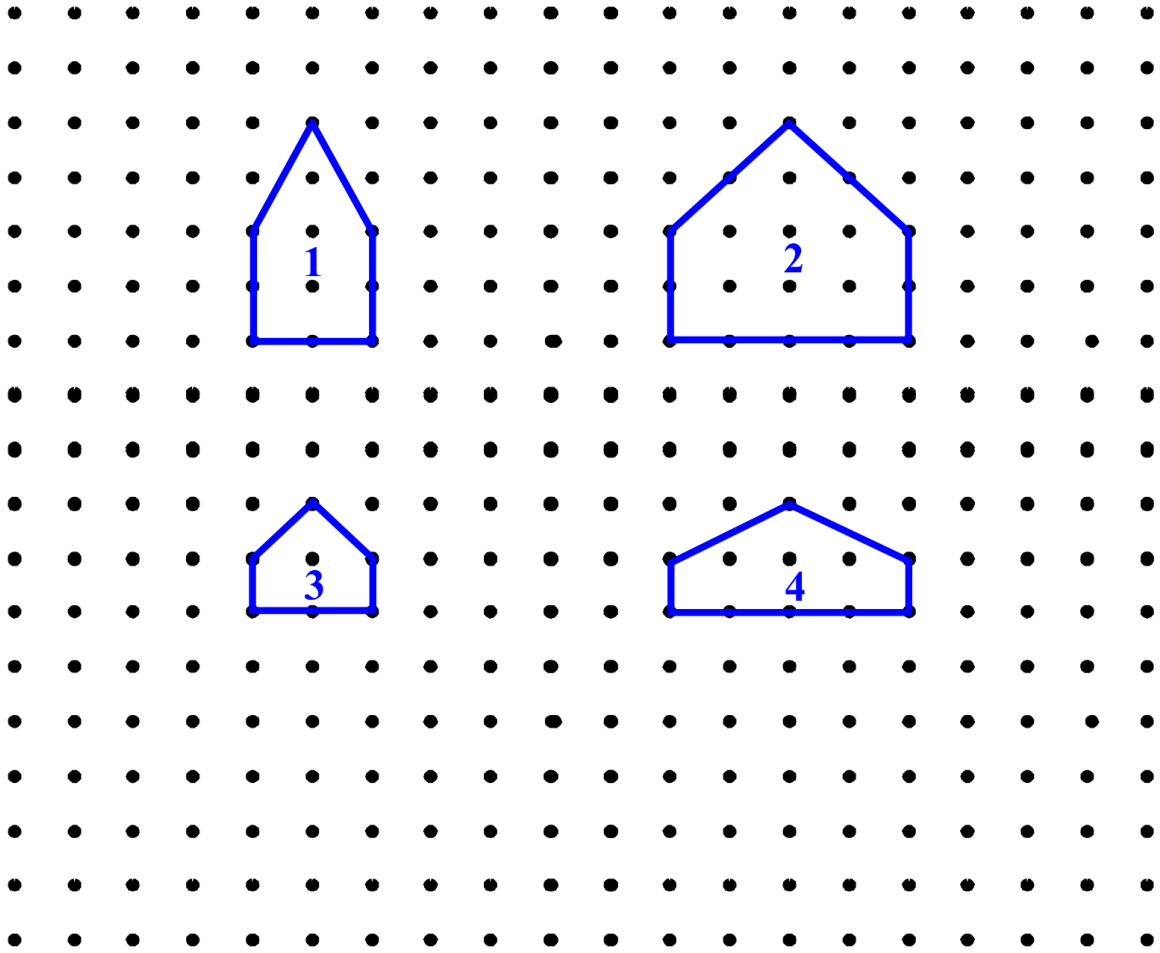
1.4.3: What Is Similarity? (continued)

6. Use the geoboards to explore whether the following triangles are similar.

a) 	Explain your reasoning.
b) 	Explain your reasoning.
c) 	Explain your reasoning.

1.4.4: Exploring Similarity

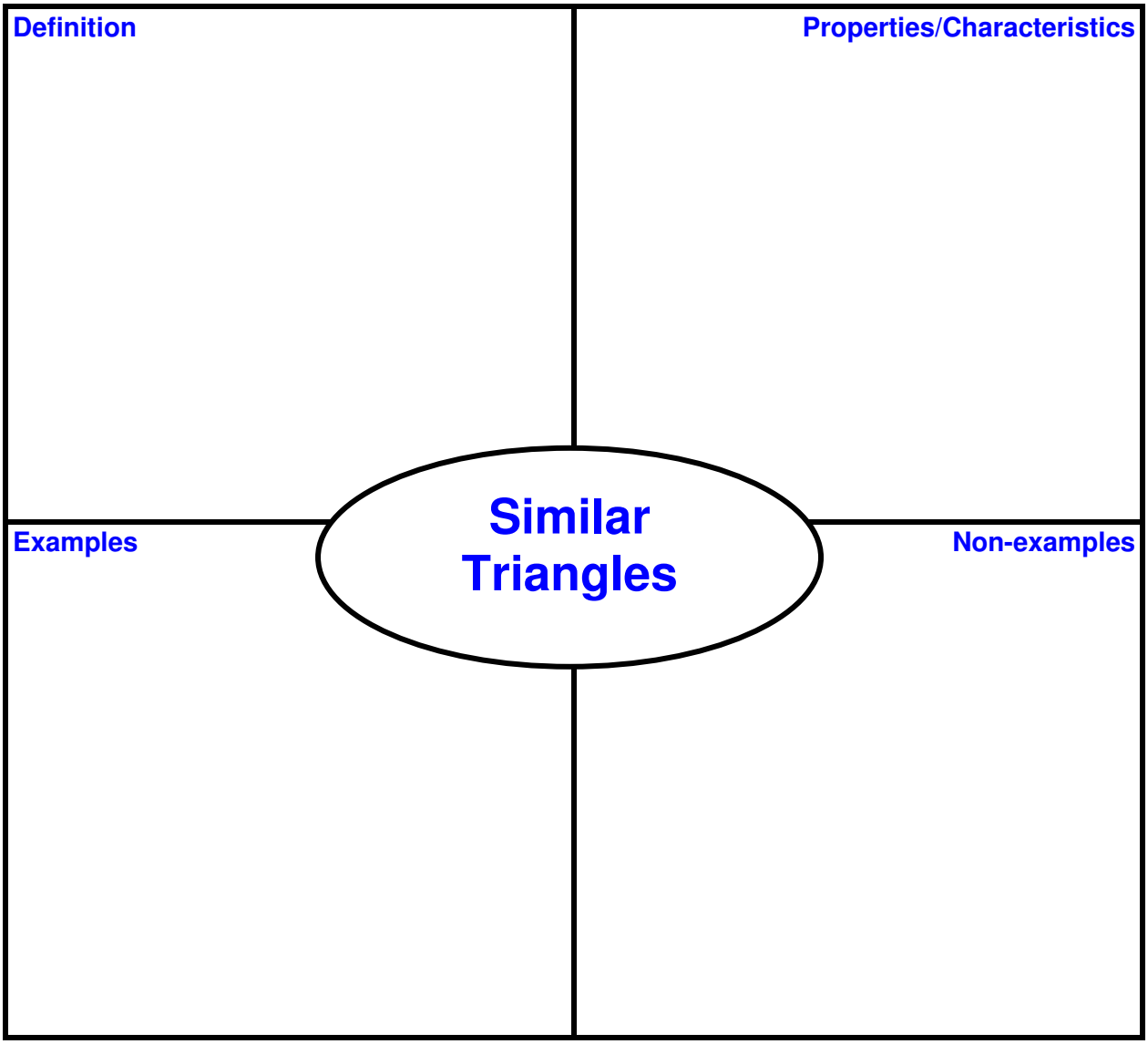
1. Which of the following four houses are similar? Explain why.
   Label the diagrams.



2. On the grid, draw a house that is similar to one of the figures.
   Complete the following statement:
   The house I drew is similar to house #_____.

   I know this because:

1.5.1: Similar Triangles



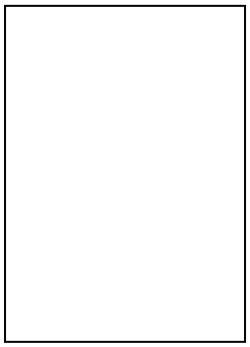
1.5.2: Finding Similar Triangles

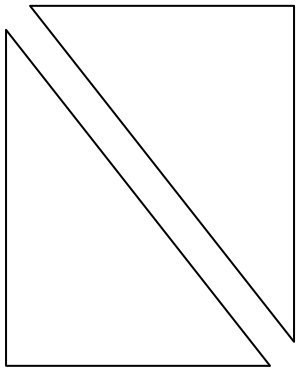
You and your partner will need:

- one sheet of legal size paper and one sheet of letter size paper.
- protractor
- ruler
- scissors

1. Measure and label the side lengths on your piece of paper. Write a large signature across the back of your piece of paper. (You may need this later.)

2. Each rectangle has two diagonals. Fold your paper along one of the diagonals. Cut the paper along the diagonal.





3. What do you notice about the two triangles that you have created?

4. Take one of the two congruent triangles and set it aside. Take the other one and using a ruler and protractor draw a line that is perpendicular to the hypotenuse and passes through the vertex of the right angle. Cut the paper along this line. You should now have three triangles.

   Label the vertices of each triangle with appropriate letters (Largest triangle is ΔABC, Middle triangle is Δ DEF, Smallest triangle is ΔGHJ.)

   Explore the relationship between the triangles by reorienting them and overlapping the three triangles so that corresponding angles are in the same place.

5. Identify any triangles that you think are similar. Explain.

1.5.2: Finding Similar Triangles (continued)

6. Using a ruler and protractor complete the table below to determine whether the triangles are similar.

Triangle	Hypotenuse	Shortest side	Middle side	Angles
ΔABC				
Δ DEF				
ΔGHJ				

7. Complete the following calculations.

$$\frac{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta DEF}{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta ABC} =$$

$$\frac{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta DEF}{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta GHK} =$$

$$\frac{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta ABC} =$$

$$\frac{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta GHK} =$$

$$\frac{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta ABC} =$$

$$\frac{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta GHK} =$$

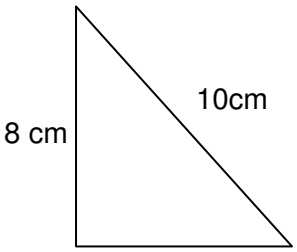
8. What do you notice about the ratios you have calculated in each column? State each ratio.
This ratio is called a scale factor.

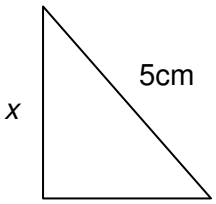
1.5.2: Finding Similar Triangles (continued)

9. What conclusions about the triangles can you draw based on the ratios calculated in question 7? Are they similar or not? Explain.

10. If you were given a triangle with side lengths specified and a scale factor how could you use this information to determine the side lengths of the similar triangle that would be created?

11. Use your method above to solve the following triangles.



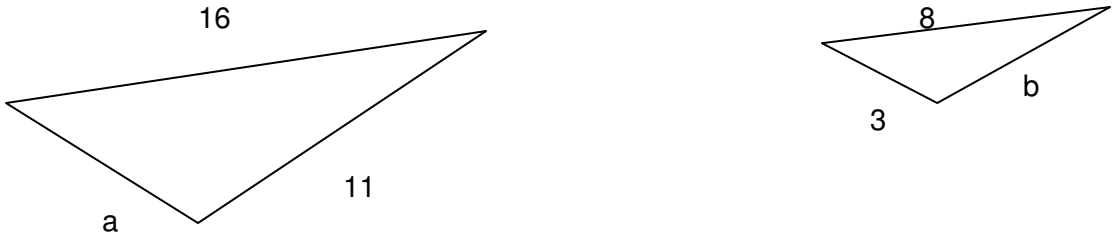


12. Try to recreate your original rectangle.

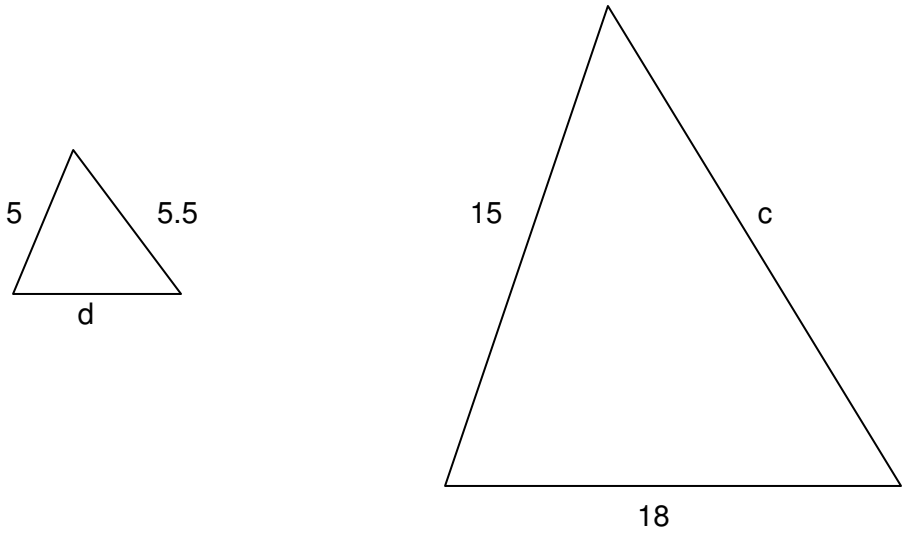
1.5.2: Similar Triangles Practice

1. Calculate the missing information for the following pairs of similar triangles.

a)



b)



1.6.1: Let's Do Proportions

1. State whether the ratios are proportional. Give reasons to support your answers.

a) $\frac{11}{12}, \frac{18}{27}$ b) $\frac{6}{102}, \frac{1}{17}$ c) $\frac{11}{8}, \frac{22}{16}$

2. Solve each proportion.

a) $\frac{2}{18} = \frac{b}{6}$ b) $\frac{a}{7} = \frac{18}{42}$ c) $\frac{2}{14} = \frac{1}{k}$

3. Solve each proportion.

a) $\frac{u}{12} = \frac{25}{10}$ b) $\frac{5}{d} = \frac{4}{6}$ c) $\frac{6}{8} = \frac{r}{9}$

4. Create a proportion from each set of numbers. Only use **four (4)** numbers from each set of numbers.

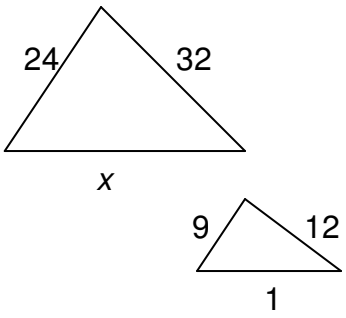
a) 21, 7, 18, 6, 14 b) 16, 2, 1, 21, 8 c) 10, 15, 20, 25, 30

1.6.2: Solving Those Proportions

1. Solve the following.

a) $\frac{3}{5} = \frac{x}{20}$ b) $\frac{x}{3} = \frac{5}{6}$ c) $1.5 : 3 = y : 10$ d) $h : 25 = 4 : 10$

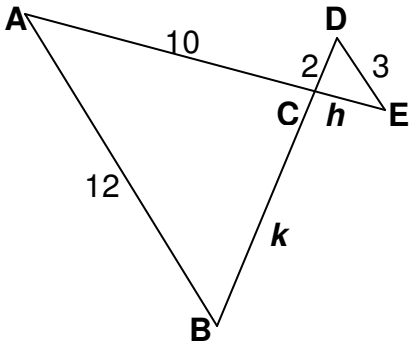
2. These are two similar triangles.



(a) Which proportion could be used to solve for *x*?

(b) Now, solve that proportion.

3. **AB** is parallel to **DE**. Solve for ***h*** and ***k***. (Hint: Redraw the triangles so that the corresponding angles are in the same position.)



1.6.3: Practice

1. **Flagpole**: The flagpole casts a shadow 14.5 m long at the same time that a person 1.8m tall casts a shadow 2.5 m long. Find the height of the flagpole. (Draw a diagram.)

2. **CN Tower**: The CN Tower casts a shadow 845.8m long. A 1.83m tall person standing near the tower casts a shadow 3.05m long. How tall is the CN Tower?

3. **Communication:** If two triangles are **similar**, explain, in your own words, what that means?

4. A triangle has sides whose lengths are 5, 12, and 13.

   A similar triangle could have sides with lengths of _____, _____,and _____.

   Another similar triangle could have sides with lengths of _____, _____,and _____. (Different than either of the others in this question).

1.7.1: How Far?

ACTIVITY 1

Your arm is about ten times longer than the distance between your eyes. Verify.

Arm length: _____ cm

Distance between eyes: _____ cm

Ratio of arm length to distance between eyes: _____ cm

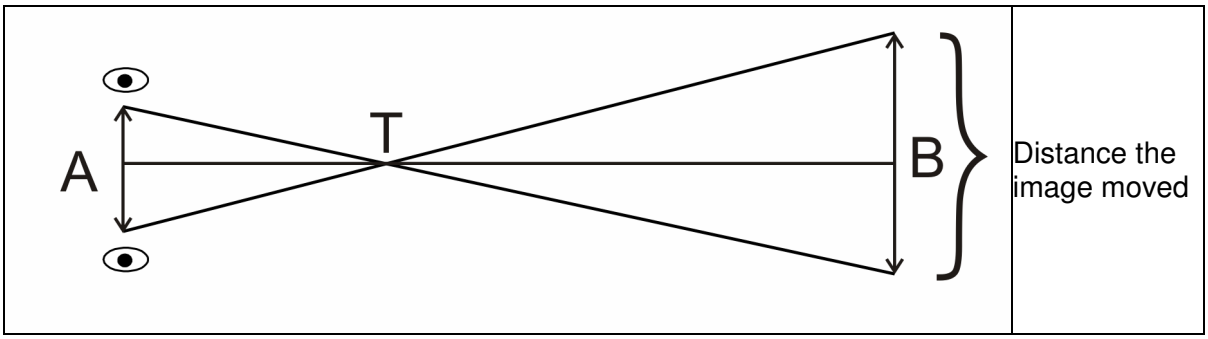
1. Select an object from which you want to determine the distance. _____ (object)

2. Estimate the width of the object. _____ cm

3. Hold one arm straight out in front of you, elbow straight, thumb pointing up. Close one eye, and align one side of your thumb with a particular spot on the front of the object. Without moving your head or arm, sight with the other eye. Your thumb will appear to jump sideways.

   a) Approximate the number of widths of the object your thumb appeared to move. _____

   b) What is the distance the image moved? _____ cm

4. 

   In the diagram:

   T is the position of your thumb.
   AT represents the length of your arm.
   TB represents the distance from your thumb to the object.

   a) Indicate all known measurements on the diagram. Include units.

   b) Identify which triangles are similar. Label the triangle vertices.
   Write the proportion needed to find the distance the object is from you.

   c) Determine the distance the object is from you, using two different methods.

1.7.2: How High? – Part 1

ACTIVITY 2

1. Select an object whose base is at right angles to the ground and whose height you cannot measure. _____(object)

2. Measure the length of the shadow of the object (and indicate units): _____

3. Hold a metre/yard stick at right angles to the ground, and measure the length of its shadow. (Use the same units as in question 2): _____

4. Draw similar triangles representing this situation in the space below. Label the diagram and indicate all known measurements with units.

5. Write the proportion needed to find the desired height.

6. Calculate the height of the object. *Show your work*.

1.7.3: How High? – Part 2

ACTIVITY 3

1. Select an object whose height you cannot measure. _____ (object)

2. Lay a small mirror horizontally on the ground exactly 1 metre in front of the object.

3. Slowly walk backwards until you can just see the top of the object in the mirror.
   Measure your distance from the mirror. _____

4. Measure the distance from the ground to your eye level. _____

5. Draw similar triangles representing this situation in the space below. Label the diagram and indicate all known measurements with units.

6. Write the proportion needed to find the desired height.

7. Calculate the height of the object. *Show your work*.

1.7.4: How High? – Part 3

ACTIVITY 4

1. Select an object whose height you cannot measure. _____

2. **Person 1:** Walk at least 20 large steps away from the object.
   Place your eye as close to the ground as possible and close your top eye. Your job will be to line up the top of the metre stick with the top of the object.

3. **Person 2:** Place the metre stick between Person 1 and the object. The metre stick must be kept at a 90˚ angle with the ground. Slowly move the metre stick towards or away from the object on the instructions of Person 1. Hold still when Person 1 has lined up the objects.

4. **Persons 3 and 4:** Measure the distance from Person 1 to the metre stick. _____ Then measure the distance from Person 1 to the object. _____

5. Draw similar triangles representing this situation in the space below. Label the diagram and indicate all known measurements with units.

6. Write the proportion needed to find the desired height.

7. Calculate the height of the object. *Show your work.*

1.8.1: Eye, eye, eye!!

Hurricanes are violent storms, which form over the warm waters of the oceans. Each year hurricanes cause millions dollars of damage when they hit coastal areas. Hurricanes can produce winds with speeds up to 241 or more kilometres per hour.

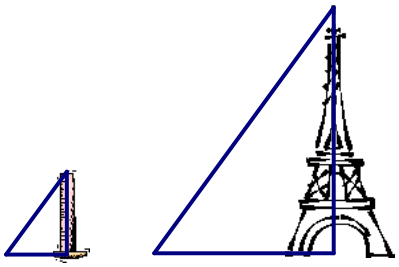
The centre of a hurricane is called the EYE. Inside the eye of a hurricane there is almost NO WIND. The air is perfectly calm and just outside the eye are the most violent winds of the storm.

How far across is the eye of this hurricane (in km)?

This photo was taken with a 90 mm camera lens on a Linhof camera at an altitude of 267 km. Draw a diagram to help.

1.8.3: Practice

1. A tower casts a shadow that is 750 m long. At the same time, a metre stick casts a shadow 1.4 m long. Label the diagram. Find the height of the tower.



2. Sam places a mirror on the ground, 5 m from the base of a tree. He then walks backwards until he can see the top of the tree in the mirror. He is now standing 0.75 m from the mirror. Sam's eye level is 1.75 m high. Label the diagram. Find the height of the tree.